The Green tensors for Stokes flow in various geometries. I. Pressure fields

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Course on Microhydrodynamics at IPPT PAN
2009/2010

The problem

Users' guide - general remarks

- WWe choose the following units:
  - Length scale: the wall-particle distance h.
  - Pressure $p_0 = \frac{F}{4\pi h^2}$.
- Dimensionless quantities:
  - Distance $r = \frac{\boldsymbol{r}_{dim}}{h}$.
  - Pressure $\mathcal{P}(\boldsymbol{r}) = p(\boldsymbol{r}_{dim})/p_0$.
- Pressure field in an infinite fluid is

$$p(\boldsymbol{r}) = \frac{\boldsymbol{F} \cdot \boldsymbol{r}}{r^3}$$

- We plot the pressure field at a point $\mathbf{r} = (x, y, z)$. Here $z = 0$, x is directed horizontally, y is directed vertically and $r = \sqrt{x^2 + y^2}$.

Users' guide - general remarks II

- The pressure sign has been marked on the plots in red for positive pressure and in blue for negative pressure.
- The coordinate ranges on every plot are identical to serve for the purpose of comparison.

$$x \in (-3.14, 3.14) \qquad y \in (-3.14, 3.14)$$

- The isobars are plotted for particular values:

$$\mathcal{P}_{iso} \in \{0, \pm 0.25, \pm 0.5, \pm 0.75, \pm 1, \pm 1.25, \pm 1.5, \pm 1.75, \pm 2\}.$$

- In the image systems, the images are plotted in gray, whereas the real point particles are plotted in black.
- To examine the effect of wall presence, we place the point force in the point $\boldsymbol{r} = (1, 0, 0)$.
- The Stokes-doublet has strength $2hF$, while all the other sources have strength F.

Users' guide - point force equations

- The Oseen pressure field in an infinite fluid for a point force $\mathbf{F} = -\mathbf{e}_y$

$$p(x,y) = -\frac{y}{r^3}.$$

For the point force acting in the x direction, we get similarly

$$p_x(x,y) = \frac{x}{r^3}.$$

- Stokes doublet field

$$p_{SD}(x,y) = -\frac{3xy}{r^5}$$

- Point force parallel to the free boundary. When the particle is at a distance h from the surface, the pressure is

$$P = p(x-h,y) + p(x+h,y)$$

- Point force perpendicular to the free boundary. For the distance h and $\mathbf{F} = \hat{\mathbf{e}}_x$, the pressure reads

$$P = -p_x(x+h,y) + p_x(x-h,y)$$

Users' guide - point force equations II

- Point force parallel to the rigid wall. When the particle is at a distance h from the wall, the pressure can be constructed as

$$\begin{aligned}\mathcal{P} &= p(x-h,y) - p(x+h,y) + 2h\frac{\partial}{\partial y}\left(\frac{x}{r^3}\right)_{\text{in } (-h,0)} = \\ &= p(x-h,y) - p(x+h,y) + 2hp_{SD}(x+h,y)\end{aligned}$$

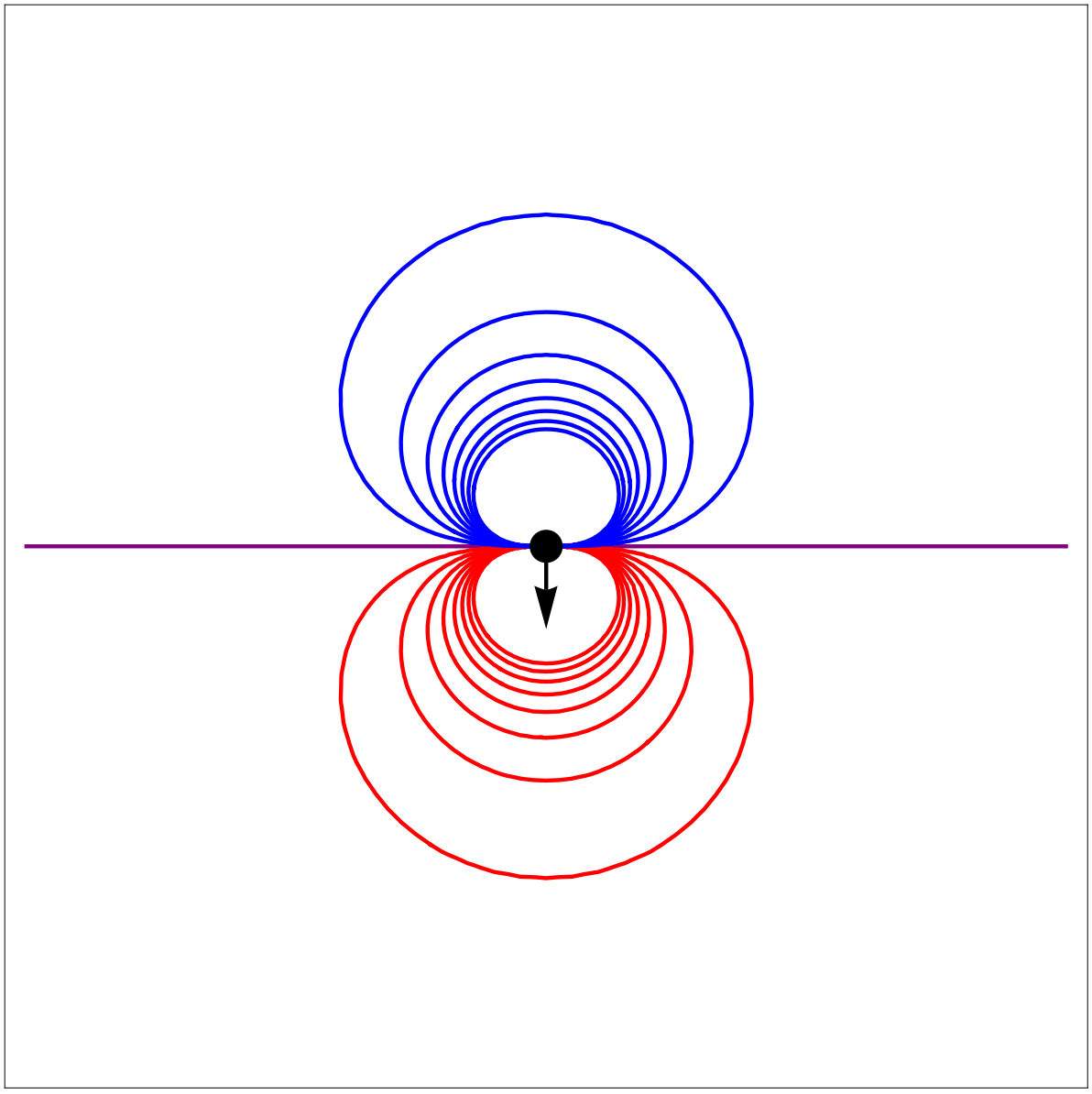
  The first two terms correspond to the particle and its image with a negative sign while the last term corresponds to the Stokes-doublet.

- Point force perpendicular to the rigid wall. When the particle is at a distance h from the wall, the pressure can be constructed as

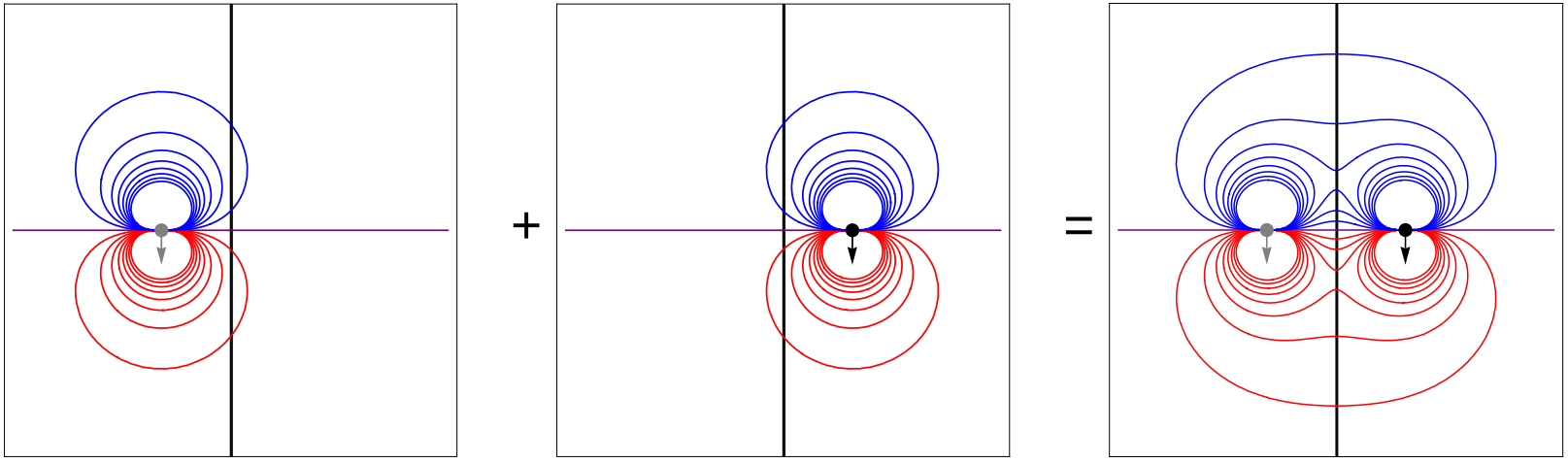
$$\begin{aligned}\mathcal{P} &= p(x-h,y) - p(x+h,y) + 2h\frac{\partial}{\partial x}\left(\frac{x}{r^3}\right)_{\text{in } (-h,0)} = \\ &= p(x-h,y) - p(x+h,y) + 2h\left(\frac{1}{r^3} - \frac{3x^2}{r^5}\right)\end{aligned}$$

  The first two terms correspond are identical as in the case of free boundary, while the last term is a Stokes-doublet in the image point $(-h,0)$ (this SD is however different form the one in the parallel force case).

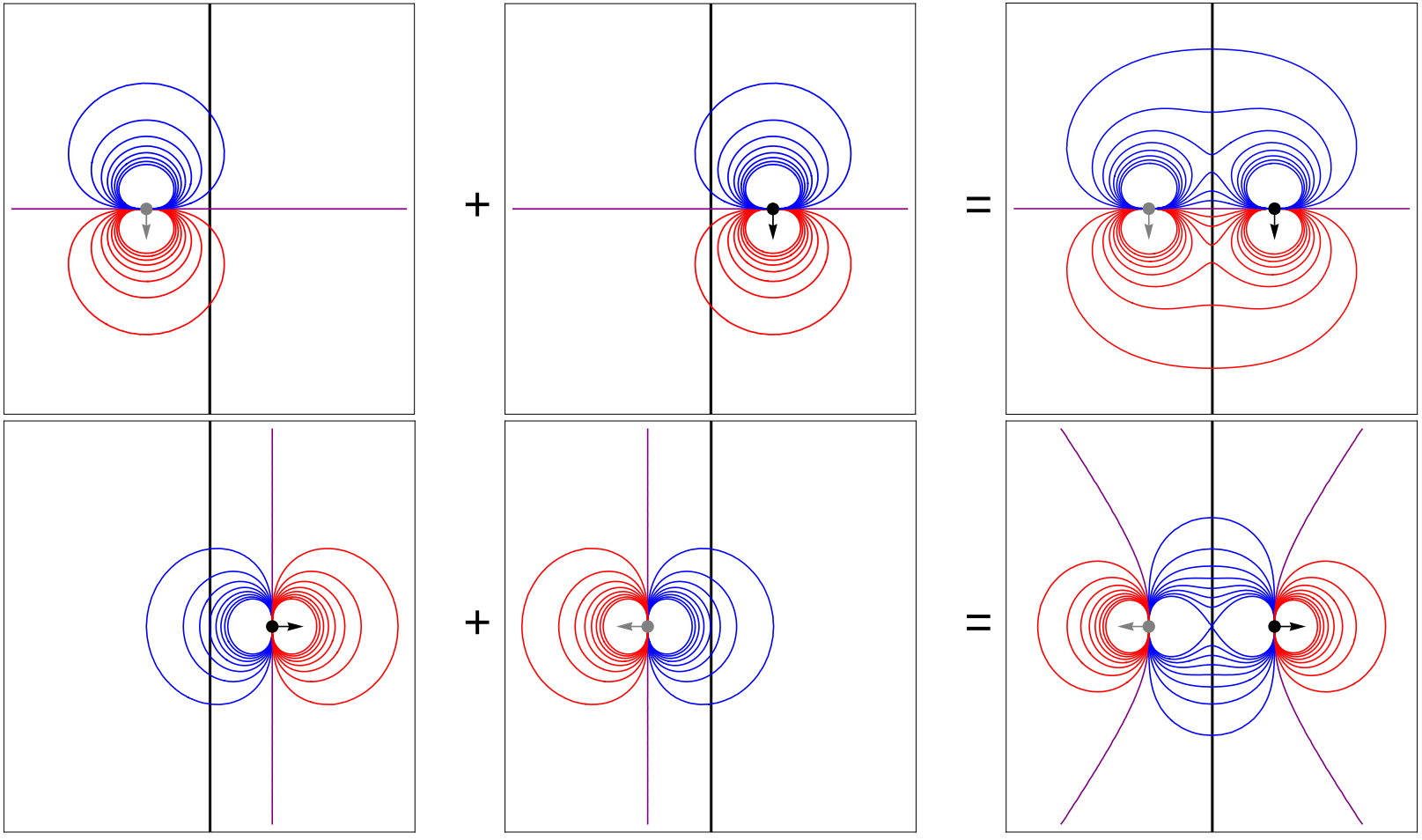
The Oseen pressure in an unbounded fluid



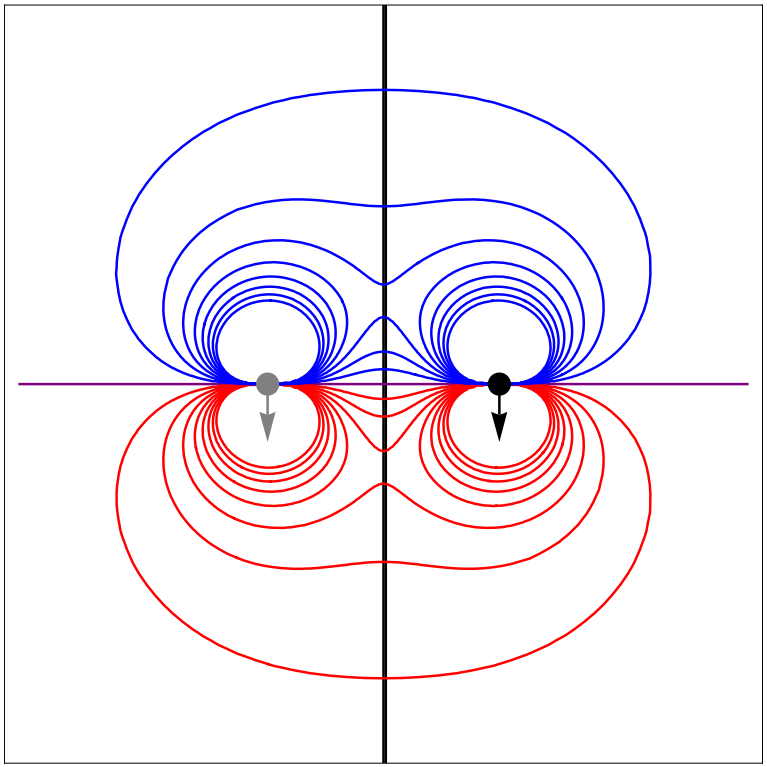
Point force near the free surface - the pressure field

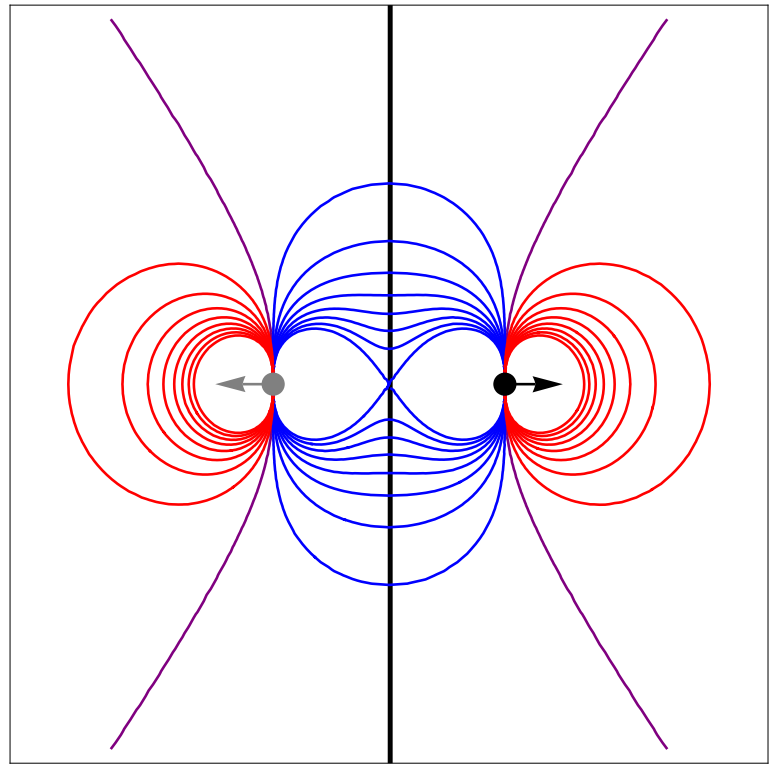


Point force near the free surface - the pressure field

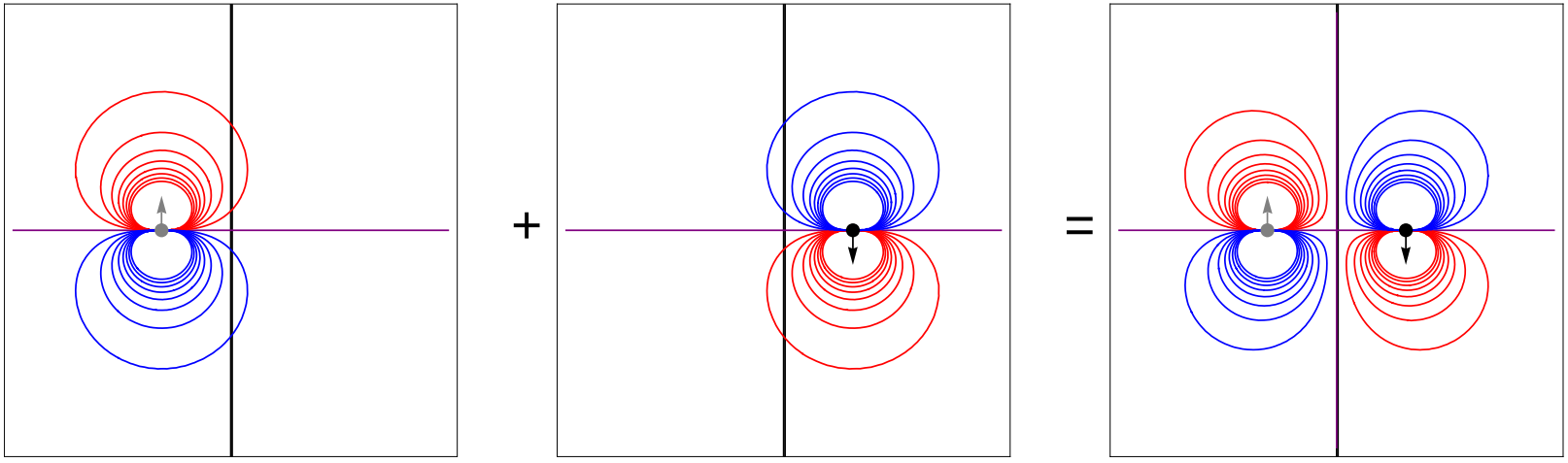


Point force near the free surface - parallel and perpendicular

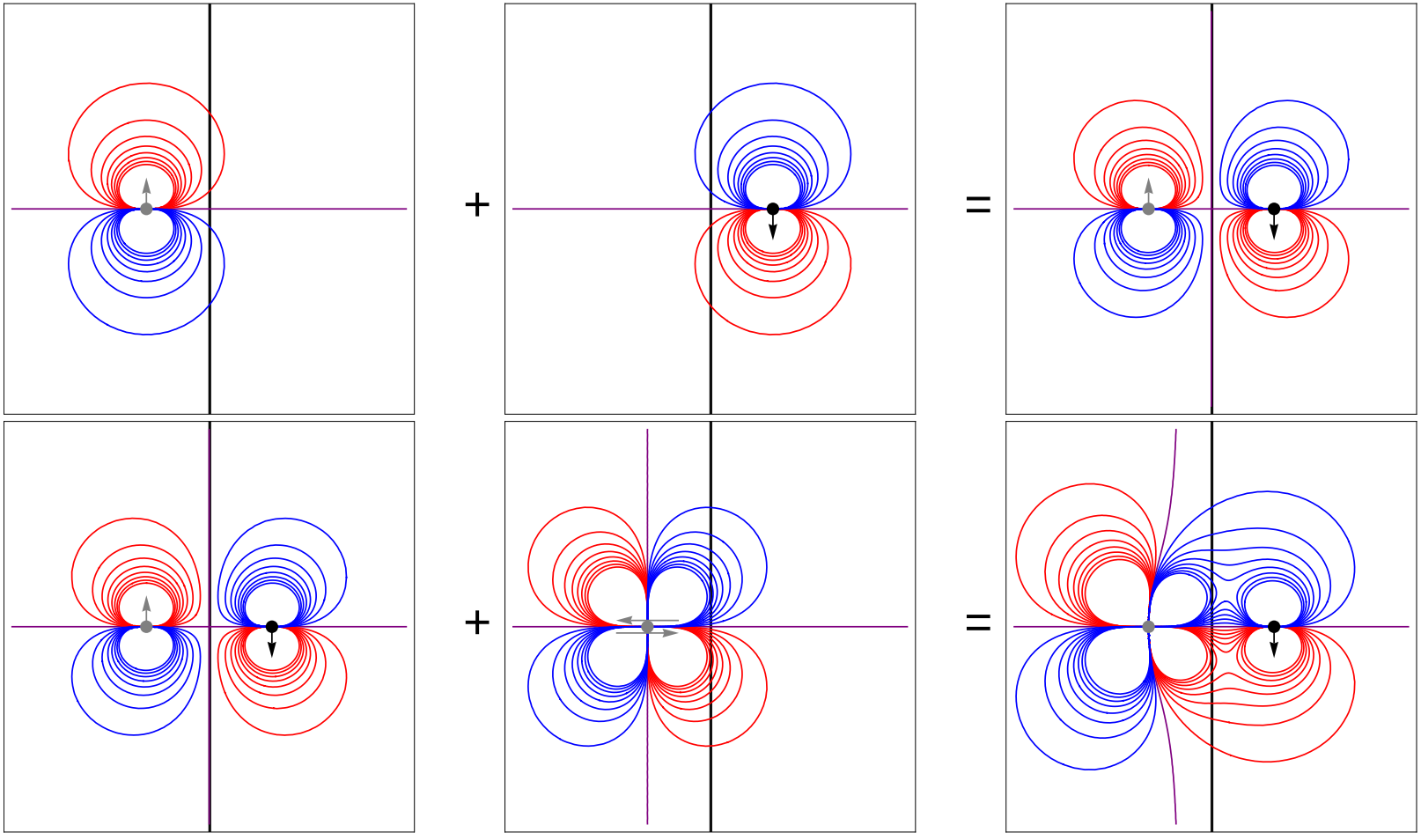




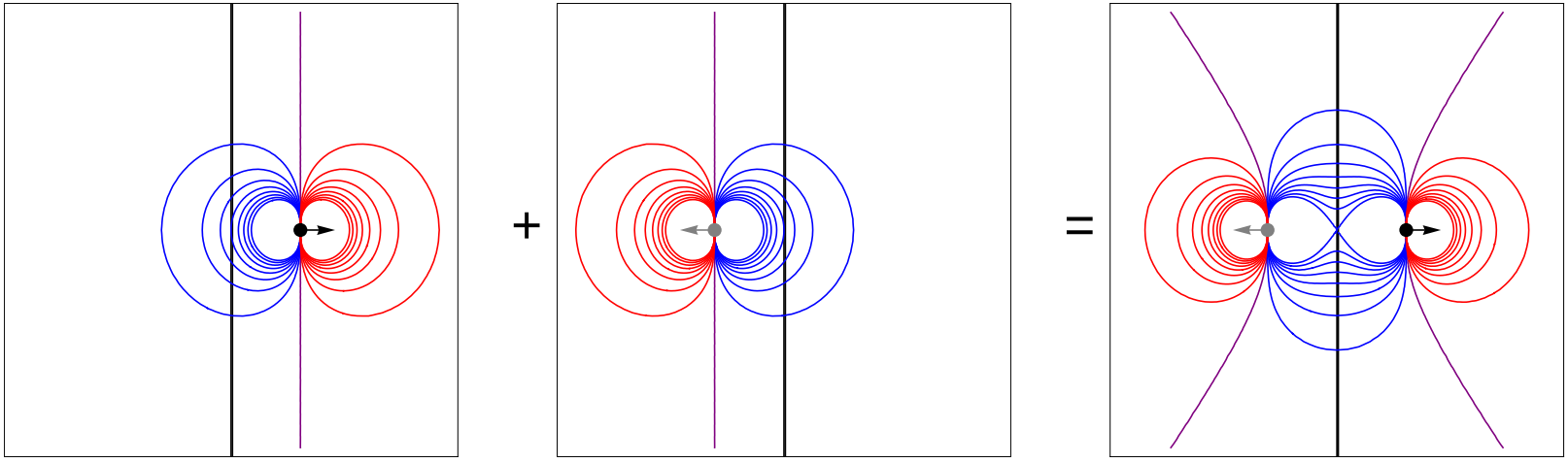
Point force near the wall - the pressure field



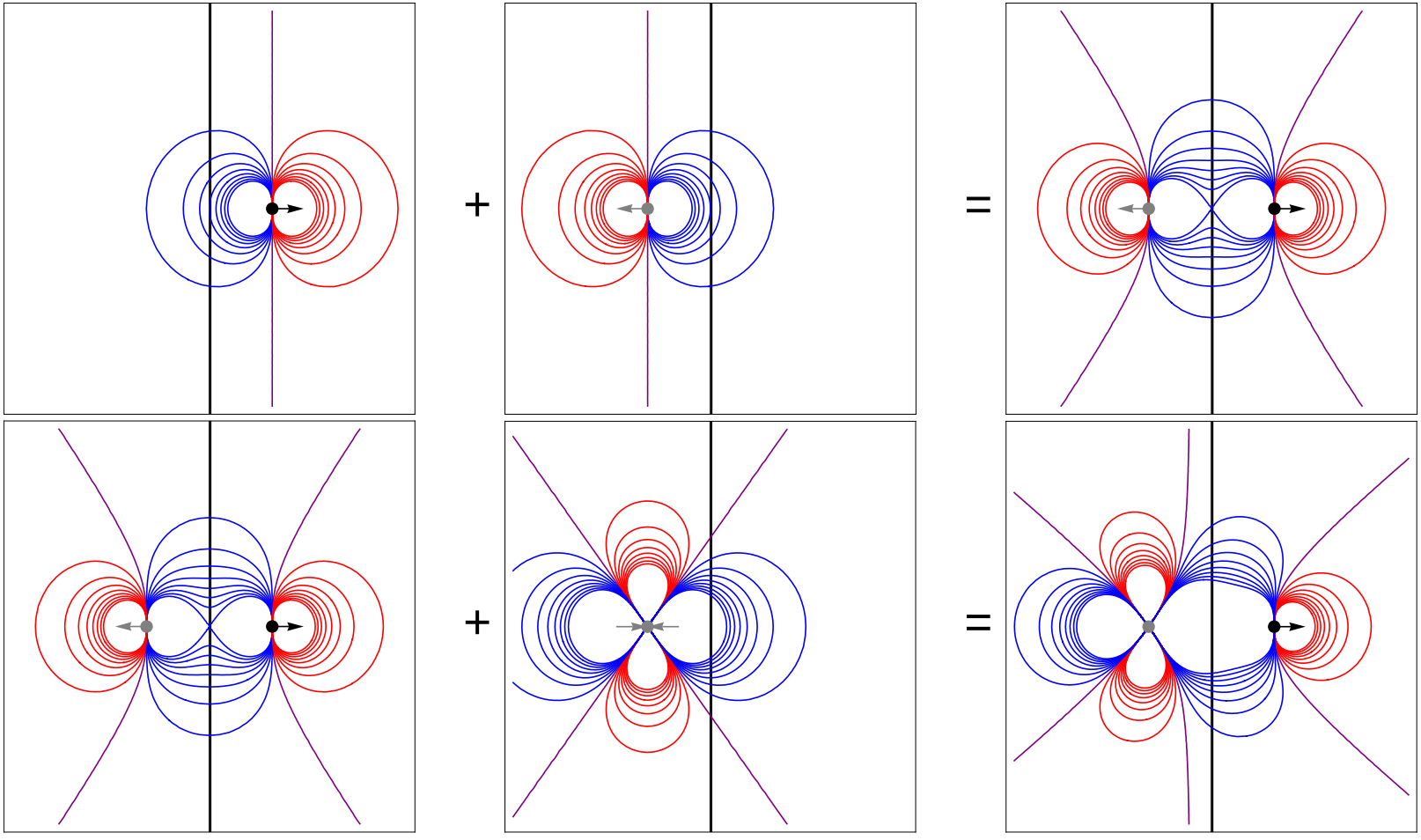
Point force near the wall - the pressure field



Point force near the wall - the pressure field



Point force near the wall - the pressure field



Point force near the free surface - parallel and perpendicular